Laplace Transforms

Lesson 18

6CT.1-4

Homework

- Problems: 3.1a,b,

3.1 a, b The following model for mean aterial pressure following an IV infusion of sodium nitorprusside (SNP) (developed by Slate and Sheppard) us

$$\Delta MAP(s) = \frac{-K_p e^{-s\delta_p}}{\tau_p s + 1} Q(s)$$

where ΔMAP is the SMP - induced change in MAP, Q is the specific SNP IV infusion rate in mg/kg patient weight/min. K_p is the patient's response constant, δ_p is the delay time in minutes to ΔMAP and τ_p is the time constant.

Let $K_p = 1.0, \tau_p = 0.75\,\text{min}, \delta_p = 0.5\,\text{min}$

a) Plot the impulse response, i.e., $Q(t) = \delta(t)$

b)Plot the response due to an unit step $U(t) = u(t)$

Homework

- Problems: 3.4a,

The unit impulse response is given as

$$h(t) = (0.7e^{-5t} + 0.2e^{-t} + 0.1e^{-0.1t})u(t)$$

Find the transfer function $H(s)$ and its poles and zeroes.

- 3.12a,b

A LTI system is described as

$$\dot{y} + 3y = x(t)$$

Find the transfer function $H(s)$ and impulse response : $h(t)$

Homework

- Problems: 13a,b

A LTI system is described as

$\ddot{y} + 8\dot{y} + 15y = 5x(t)$

Find the transfer function $H(s)$ and impulse response : $h(t)$

- Sketch $f(t) = tu(t)u(1\text{-}t)$ and find $F(s)$
- Find $f(t)$, if $F(s) = (1-e^{-2s})/s^2$. Repeat for $F(s) = (1-s+e^{-2s})/s^3$
- 6CT.2.1
- 6CT.2.2

Homework Answers #1

- Problems: 3.1a

$$\frac{\Delta MAP}{Q}(s) = \frac{-K_p e^{-s\delta_p}}{\tau_p s + 1} = \frac{-1e^{-s.5}}{.75s+1}$$

$$= -\frac{4}{3}\frac{e^{-s.5}}{s+4/3}$$

Note:

$$\pounds[f(t)] = F(s)$$

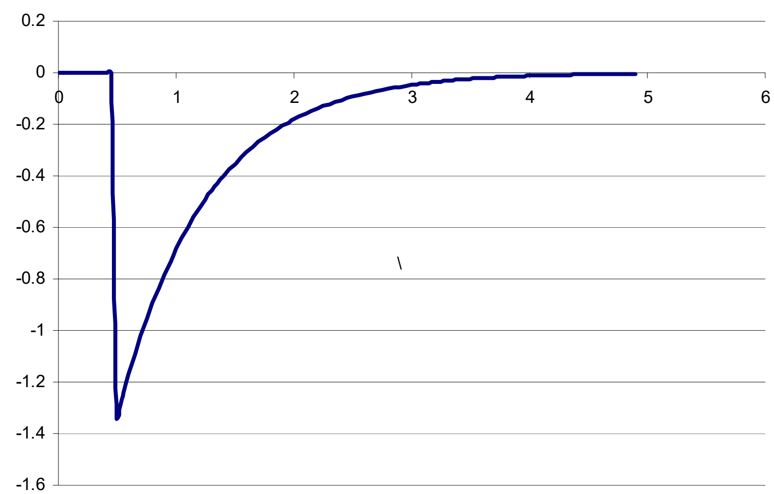
$$\pounds[f(t)e^{s_1 t}] = F(s - s_1)$$

And

$$\pounds[f(t)u(t)] = F(s)$$

$$\pounds[f(t-T)u(t-T)] = F(s)e^{-sT}$$

$$\pounds^{-1}[-\frac{4}{3}\frac{e^{-s.5}}{s+4/3}] = -\frac{4}{3}e^{-\frac{4(t-.5)}{3}}u(t-.5)$$



Homework Answers #2

- Problems: 3.1b

$$\frac{\Delta MAP}{Q}(s)=\frac{-K_p e^{-s\delta_p}}{\tau_p s+1}=\frac{-1e^{-s.5}}{.75s+1}$$

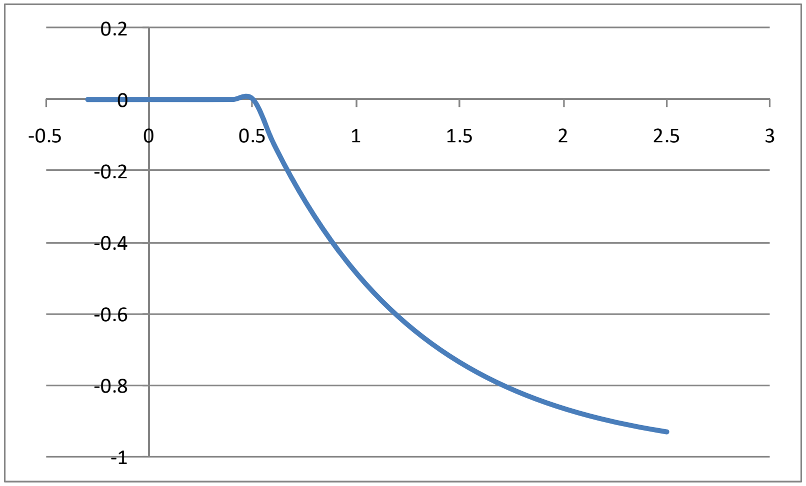
$$\frac{\Delta MAP}{Q}(s)\times\frac{1}{s}=-\frac{4}{3}\frac{e^{-s.5}}{s+4/3}\times\frac{1}{s}=-\frac{4}{3}\frac{e^{-s.5}}{s(s+4/3)}=-\frac{4e^{-s.5}}{3}\frac{1}{s(s+4/3)}$$

$$=-\frac{4e^{-s.5}}{3}[\frac{A}{s}+\frac{B}{(s+4/3)}]=-\frac{4e^{-s.5}}{3}[\frac{A(s+4/3)+Bs}{s(s+4/3)}]$$

$$\therefore A=\frac{3}{4};B=-\frac{3}{4}$$

$$=-\frac{4e^{-s.5}}{3}[\frac{\frac{3}{4}}{s}-\frac{\frac{3}{4}}{(s+4/3)}]=[\frac{e^{-s.5}}{s+4/3}-\frac{e^{-s.5}}{s}]$$

$$\pounds^{-1}[\frac{e^{-s.5}}{s+4/3}-\frac{e^{-s.5}}{s}]=e^{-\frac{4(t-.5)}{3}}u(t-.5)-u(t-.5)$$



Homework Answers #3

- Problems: 3.4a

$$h(t) = (0.7e^{-5t} + 0.2e^{-t} + 0.1e^{-0.1t})u(t)$$

$$H(s) = \frac{0.7}{s+5} + \frac{0.2}{s+1} + \frac{0.1}{s+.1}$$

$$= \frac{0.7(s+1)(s+.1) + 0.2(s+5)(s+.1) + .1(s+5)(s+1)}{(s+5)(s+1)(s+.1)}$$

$$= \frac{0.7(s^2+1.1s+.1) + 0.2(s^2+5.1s+.5) + .1(s^2+6s+5)}{(s+5)(s+1)(s+.1)}$$

$$= \frac{(s^2+2.4s+.67)}{(s+5)(s+1)(s+.1)} = \frac{(s+.32)(s+2.07)}{(s+5)(s+1)(s+.1)}$$

Homework Answers #4

- Problems: 3.12a,b

$$a)\dot{y}(t) + 3y(t) = x(t)$$

$$£[\dot{y}(t) + 3y(t)] = £[x(t)]$$

$$sY(s) + 3Y(s) = X(s)$$

$$(s+3)Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+3}$$

$$b)h(t) = e^{-3t}u(t)$$

Homework Answers #5

- Problems: 3.13a,b

$$a)\ddot{y}(t)+8\dot{y}(t)+15y(t)=5x(t)$$

$$£[\ddot{y}(t)+8\dot{y}(t)+15y(t)]=£[5x(t)]$$

$$(s^2+8s+15)Y(s)=5X(s)$$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{5}{(s^2+8s+15)}=\frac{5}{(s+3)(s+5)}$$

$$=\frac{A}{(s+3)}+\frac{B}{(s+5)}=\frac{A(s+5)+B(s+3)}{(s+3)(s+5)}$$

$$A+B=0;\ \ A=-B$$

$$5A+3B=5;\ \ 5A-3A=5; A=\frac{5}{2}; B=-\frac{5}{2}$$

$$H(s)=\frac{5}{2}[\frac{1}{(s+3)}-\frac{1}{(s+5)}]$$

$$b)h(t)=\frac{5}{2}[e^{-3t}-e^{-5t}]u(t)$$

Homework Answers #6

- Sketch *f(t) = tu(t)u(1-t)* and find *F(s)*

$$F(s)=\int_0^\infty tu(t)u(1-t)e^{-st}dt$$

$$=\int_0^1 te^{-st}dt$$

$$u=t;dt=1dt$$

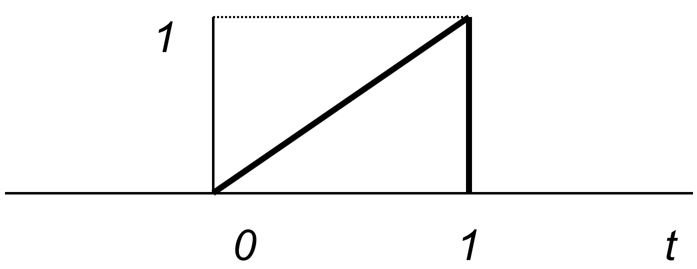
$$dv=e^{-st}dt;v=-\frac{1}{s}e^{-st}$$

$$=-\frac{1}{s}te^{-st}\Big|_0^1-\int_0^1(-\frac{1}{s})e^{-st}dt$$

$$=-\frac{1}{s}1e^{-s1}--\frac{1}{s}0e^{-s0}-(-\frac{1}{s})^2e^{-st}\Big|_0^1$$

$$=-\frac{1}{s}e^{-s}-\{\frac{1}{s^2}(e^{-s}-1)$$

$$=\frac{1}{s^2}(1-e^{-s}-se^{-s})$$



OR

$$f(t)=tu(t)u(1-t)$$

$$=tu(t)-(t-1)u(t-1)-u(t-1)$$

$$F(s)=\frac{1}{s^2}-\frac{e^{-s}}{s^2}-\frac{e^{-s}}{s}$$

Homework Answers #7

- Find f(t), if $F(s) = (1\text{-}e^{-2s})/s^2$. Repeat for $F(s) = (1\text{-}s+e^{-2s})/s^3$

$$F(s) = \frac{1}{s^2}(1 - e^{-2s})$$

Note:

$$£[tu(t)] = \frac{1}{s^2}$$

And

$$£[f(t)u(t)] = F(s)$$

$$£[f(t\text{-}T)u(t\text{-}T)] = F(s)e^{-sT}$$

$$£^{-1}[\frac{1}{s^2}(1 - e^{-2s})] = £^{-1}[\frac{1}{s^2}] - £^{-1}[\frac{e^{-2s}}{s^2}]$$

$$f(t) = tu(t) - (t-2)u(t-2)$$

$$F(s) = \frac{1}{s^3}(1 - s + e^{-2s})$$

Note:

$$£[\frac{1}{2}t^2u(t)] = \frac{1}{s^3}$$

And

$$£[f(t)u(t)] = F(s)$$

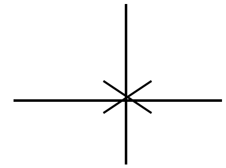
$$£[f(t\text{-}T)u(t\text{-}T)] = F(s)e^{-sT}$$

$$£^{-1}[\frac{1}{s^3}(1 - s + e^{-2s})] = £^{-1}[\frac{1}{s^3}] - £^{-1}[\frac{1}{s^2}] + £^{-1}[\frac{e^{-2s}}{s^3}]$$

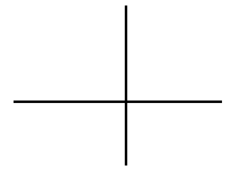
$$f(t) = \frac{t^2}{2}u(t) - tu(t) + \frac{(t-2)^2}{2}u(t-2)$$

6CT2.1

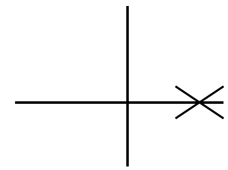
$$X(s)=\int_0^\infty x(t)e^{-st}dt$$



$$a)x(t)=3; X(s)=\int_0^\infty 3e^{-st}dt=\frac{3}{-s}e^{-st}\Big|_0^\infty=\frac{3}{-s}[e^{-s\infty}-e^{-s0}]=\frac{3}{s};\text{ Region }=\sigma>0$$

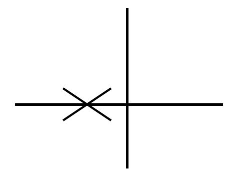


$$b)x(t)=7\delta(t); X(s)=\int_0^\infty 7\delta(t)e^{-st}dt=7;\text{ Region all }\sigma$$



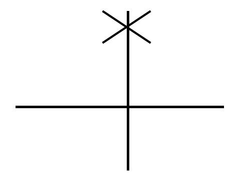
$$c)x(t)=e^{5t}; X(s)=\int_0^\infty e^{5t}e^{-st}dt=\int_0^\infty e^{-(s-5)t}dt=\frac{1}{-(s-5)}e^{-(s-5)t}\Big|_0^\infty=\frac{1}{-(s-5)}[e^{-(s-5)\infty}-e^{-(s-5)0}]$$

$$=\frac{1}{(s-5)};\text{ Region }=\sigma>5$$



$$d)x(t)=e^{-2t}; X(s)=\int_0^\infty e^{-2t}e^{-st}dt=\int_0^\infty e^{-(s+2)t}dt=\frac{1}{-(s+2)}e^{-(s+2)t}\Big|_0^\infty=\frac{1}{-(s+2)}[e^{-(s+2)\infty}-e^{-(s+2)0}]$$

$$=\frac{1}{(s+2)};\text{ Region }=\sigma>-2$$



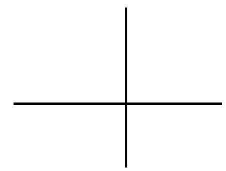
$$d)x(t)=e^{j8t}; X(s)=\int_0^\infty e^{j8t}e^{-st}dt=\int_0^\infty e^{-(s-j8)t}dt=\frac{1}{-(s-j8)}e^{-(s-j8)t}\Big|_0^\infty=\frac{1}{-(s-j8)}[e^{-(s-j8)\infty}-e^{-(s-j8)0}]$$

$$=\frac{1}{(s-j8)};\text{ Region }=\sigma>0$$

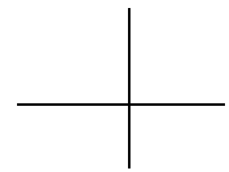
6CT2.2

$$X(s)=\int_0^{\infty} x(t)e^{-st}dt$$

$$a)\,x(t)=\delta(t-6);\,X(s)=\int_0^{\infty}\delta(t-6)e^{-st}dt=e^{-s6};\ \text{Region all }\sigma$$

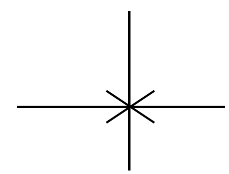


$$b)\,x(t)=\delta(t+3)+\delta(t-6);\,X(s)=\int_0^{\infty}\delta(t+3)+\delta(t-6)e^{-st}dt=e^{s3}+e^{-s6};\ \text{Region all }\sigma$$



$$c)\,x(t)=u(t-6);\,X(s)=\int_0^{\infty}u(t-6)e^{-st}dt=\int_6^{\infty}e^{-st}dt=\frac{1}{-s}e^{-st}\Big|_6^{\infty}=\frac{1}{-s}[e^{-s\infty}-e^{-s6}]$$

$$=\frac{1}{s}e^{-s6};\ \text{Region}=\sigma>0$$



$$d)\,x(t)=u(t+3);\,X(s)=\int_0^{\infty}u(t+3)e^{-st}dt=\int_{-3}^{\infty}e^{-st}dt=\frac{1}{-s}e^{-st}\Big|_{-3}^{\infty}=\frac{1}{-s}[e^{-s\infty}-e^{s3}]$$

$$=\frac{1}{s}e^{s3};\ \text{Region}=\sigma>0$$